CS 202 Fall 2014
Exam #2, 3 November

Name: ______________________________

You have 70 minutes for this exam.

Read briefly through the whole exam before you start.

Don't spend too much time on any one question.

No notes, calculator, etc. are allowed.

Except where otherwise indicated, always show your work or otherwise explain your answer. A correct answer with no supporting argument may not earn much credit.

That said, do your scratch work on a separate piece of paper.

Carefully indicate your answers — draw boxes around them to distinguish them from your other work. Please write legibly.

If you think that a question is unclear or ambiguous (or if you think that there is an error), make a good-faith effort to interpret the intent of the question, and explain your interpretation in your solution.

Good luck!

Question:	1	2	3	4	5	6	Total
Points:	8	12	5	5	9	6	45
Score:							

(8 pts.) 1. Tell me the tight asymptotic upper bound (big O), in simplest terms, of the following functions. You don't have to prove your answers.

(a) $n^3 + 2n - 1$

(c) $4n + 2n\log_2(n) + 8\log_2(n)$

(b) $5n^6 + 4 \cdot 2^n - \frac{1}{2}n^2$

(d) $\sqrt{10n}\log_2(n) + 2(n-1)$

2. For these questions, if the answer can be expressed straightforwardly using the $P(n,k)$ or $C(n,k)$ or $\binom{n}{k}$ notation defined in class, then that is a required part of the answer. You must also to write out your answer in terms of factorials or other high-level mathematical operations, whether or not you use one of the concise notations above. There's no need, however, to compute a number for your answer.

(4 pts.) (a) There are 17 items available at the cafeteria, and you want to get exactly 4 distinct items for lunch. How many different meals could you make in this way?

(4 pts.) (b) Your newly-founded island nation needs a government. You and your seven friends (that's a total of 8 people) are the only ones on the island, but your fledgling government has 15 distinct positions in it, from President down to Undersecretary of Poultry Education. Given that any person can be named to any number of positions (including zero), how many different possible configurations of your government are there?

(4 pts.) (c) Your website design needs 5 distinct colors on it, each for a different purpose: background, foreground, text color, link color, and accents. Through intensive market research and numerical simulation, you've prepared a palette of 15 possible colors to choose from. How many different choices can you make for the combination of colors in your design?

(5 pts.) 3. Prove that $\frac{1}{2}n^2 - 2 = \Omega(n^2)$. *[Plan your proof out on scratch paper first!]*

(5 pts.) 4. Prove the following equality: $\binom{n}{k-1}\frac{n-k+1}{k} = \binom{n}{k}$. *[Plan your proof out on scratch paper first!]*

5. To the right is the adjacency matrix of a graph G, in which the vertices are numbered consecutively starting from 1.

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(2 pts.) (a) What is the degree of node 2?

(1 pt.) (b) Is it possible that G is undirected? (Answer yes or no.)

(3 pts.) (c) What distance would breadth-first search, started from node 4, assign to node 5?

(3 pts.) (d) Tell me a simple cycle in G. (Just listing the nodes in order is fine.)

6. For this question, you have the matrices A, B, and C defined to the right.

$$A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & -3 \\ -1 & 2 & 1 \\ 2 & -3 & 3 \end{bmatrix}$$

(1 pt.) (a) Is the matrix AB well-defined? If so, how many rows does it have?

(1 pt.) (b) Is the matrix CA well-defined? If so, how many columns does it have?

(4 pts.) (c) Is the matrix BC well-defined? If so, write it out here: